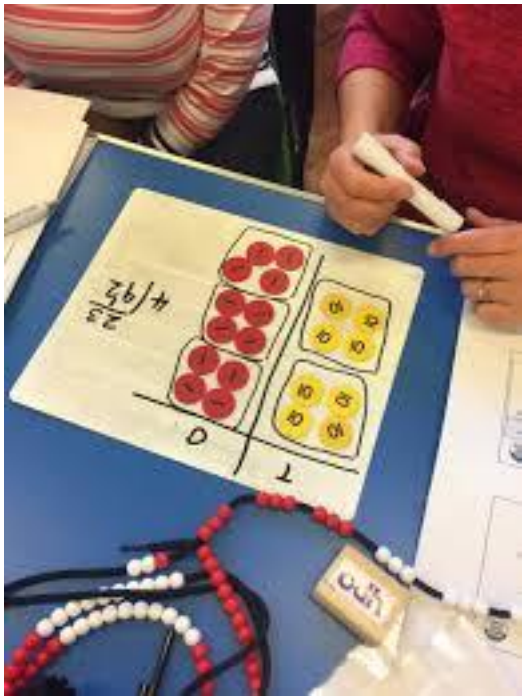
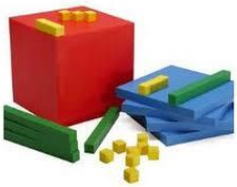




West Somerset Calculation Policy Summer 2018

This policy has been designed to teach children to develop conceptual understanding through the progression of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

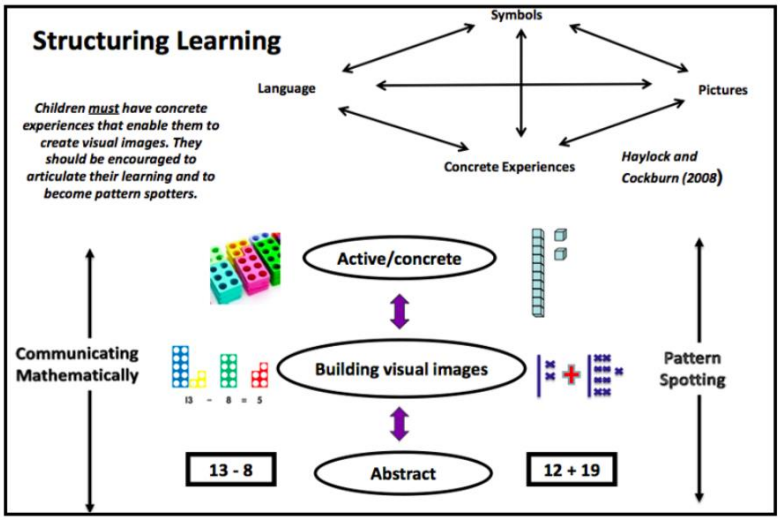
Background
This policy has been developed by CLP Maths Coordinators (Primary/Secondary).

Using the concrete-pictorial-abstract approach:
Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

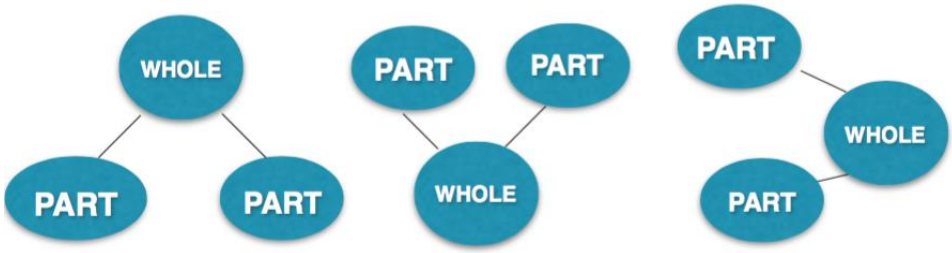
Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 .



Part/Whole Model – Key Structures

Addition and Subtraction are connected. Add parts together to equal the whole, whole subtract part to name the missing part.



Guidance

This is document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified.

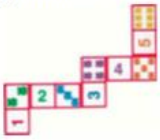
Early Years

Developing Number Sense

Vocabulary

Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on.
equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Ordinality:

Concrete:	Pictorial:	Abstract:
Children place a range of physical dominoes in a set order. 	Children match representations in a set order, for example, using pictorial bear / number dominoes. 	Children fill in spaces on a partially filled number track and create representations to show different totals (extension) – helping pupils to make the transition from understanding ordinality to cardinality.

1		3	4		6	7	8		10

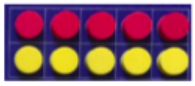
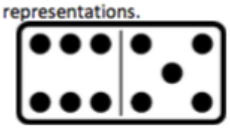
Ordinal numbers:

Concrete:	Pictorial:	Abstract:
Children physically line up ducks in a row and verbally label them, e.g. 'first /second / third.' 	Children order slides with pictures of ducks, for example, on the Interactive Whiteboard. 	Children apply their understanding of ordinal numbers, e.g. by using written 1st, 2nd and 3rd labels and other related verbal language when ordering objects. 

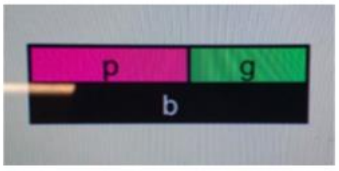
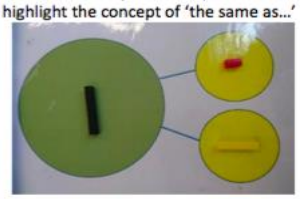
Cardinality:

Concrete:	Pictorial:	Abstract:
Children use a range of structured and unstructured apparatus, plus natural resources, to create different number values. 	Children recognise different number values that are presented in pictorial forms. 	Children are asked a range of questions that allow them to show an application of understanding related to cardinality, e.g. Can you find a collection of…[objects]…to represent six? Can you show me six fingers?

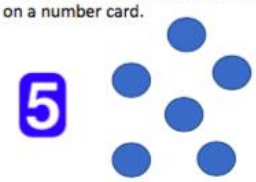
Subitising:

Concrete:	Pictorial:	Abstract:
Children replicate a range of physical representations, which they then verbally interpret without a need to count objects.  	Children use picture prompts to practise their recognition of number representations.  	Children use finger paint to show various 1-6 representations. 

Equality:

Concrete:	Pictorial:	Abstract:
Children use physical equipment when learning about equality (also inequality), and also use related language, e.g. 'the same as,' 'more than' and less than.'      	Children use pictorial representations to show equality or values that are 'the same as,' whilst also verbalising their reasoning, e.g. 'pink and green are the same as black…' 	Children use the cherry model to record either written numerals or pictorial representations that highlight the concept of 'the same as…'  

1 to 1 correspondence:

Concrete:	Pictorial:	Abstract:
Children count various physical objects by partitioning a group and finally recombining. 	Children count the dots on the face of a pictorial dice. 	Children draw dots to match the number of holes that can be seen on a named Numicon shape. 
Children write a number in each part of a muffin tin and then put the appropriate number of buttons in each section. 	Children match number cards to pictures of the equal numbers of buttons. 	Children cut out buttons equal to the number shown on a number card. 

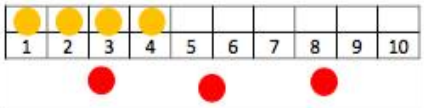
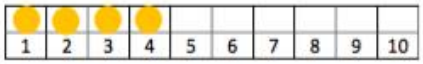
Conservation of number:

Concrete:	Pictorial:	Abstract:
Children explore whether the number of cubes stay the same or change when they are moved within a shape.  Pupils also count dolls and then put them in different rooms before re-counting to check the total. Hopefully they decide that if nobody has left and nobody has arrived, then it must be the same total even if some of the dolls have moved rooms. 	Pupils work with visual reminders of their concrete experiences – to check how their understanding around conservation of number has changed.  	Children are provided with opportunities to further explore and prove their thinking. They may be asked to put a total of dolls in the toy house and then move them around. In order to prove it is still the same total, they can take the dolls and put them onto a number track, whilst also applying their understanding about the cardinal principle.  

Concept of zero:

Concrete:	Pictorial:	Abstract:
Children use a shuffle box with up to ten objects in. After the box has been shaken, pupils write out the corresponding number sentence, e.g. 2 = 1 + 1, depending on where the objects have landed. Query what happens if there is nothing on one side. Introduce to children the concept of zero, e.g. 2 = 2 + 0. 	Children use pictorial representations to see that you can have an amount that's called 'zero.' Pupils are required to count the number of apples of a tree, and circle the trees which have no apples. 	Children can be encouraged to represent written number sentences by creating visual shuffle boxes using finger paint, e.g. 5 = 0 +5  Pupils should be able to grasp the concept of zero to use within number sentences, e.g. 4 = 4 + 0 … and verbalise … "I know that four is the same as four add zero."

Counting on:

Concrete:	Pictorial:	Abstract:
Children use physical objects to learn the skill. For example, they count on from the larger value by using their fingers whilst pointing at each 'extra' dot on the second side of a domino.  In addition, pupils use counters on number tracks to rehearse the process of counting on. 	Children use a die to generate numbers and count on from pictorial representations of counters already positioned on a number track. 	Children apply their understanding of this skill by playing games such as 'snakes and ladders.' 

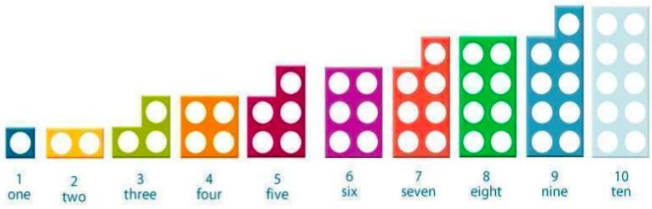
Reception

Addition

Vocabulary

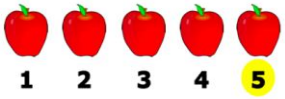
Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on.

Use of **Numicon** is another great way to help children develop mental representations of number.



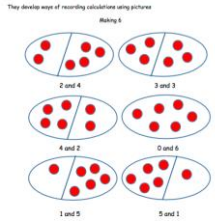
These experiences and number representations will help children:

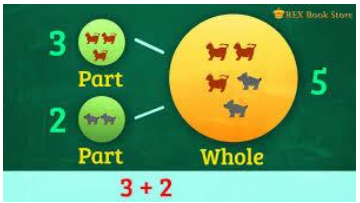
- **Reliably count the number of objects in a set using the numbers one to twenty.**

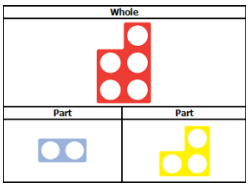


- **Say which number is one more or one less than a given number.**

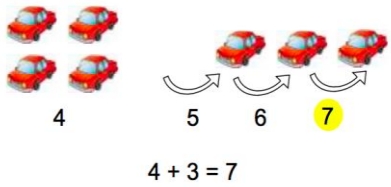
Explore part /whole relationship

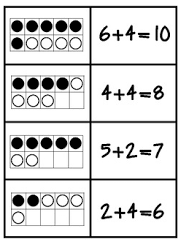


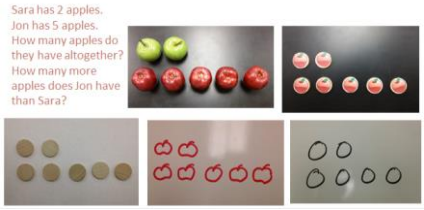




- **Use objects to add two single-digit numbers by counting on to find the answer.**







Solving problems using concrete and pictorial images.

Subtraction

Vocabulary

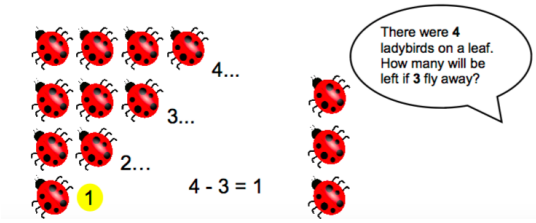
Part, whole, equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

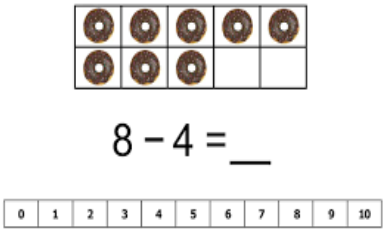
- **Use objects to subtract two single-digit numbers by counting back to find the answer.**

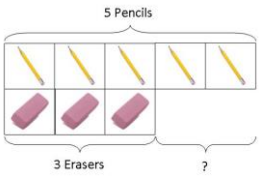
The first step into **subtraction** is to learn how to count **backwards**.

Let's count backwards from 14!

Children will then utilise this strategy to solve simple subtractions:







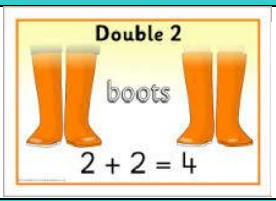
Solving problems using concrete and pictorial images.

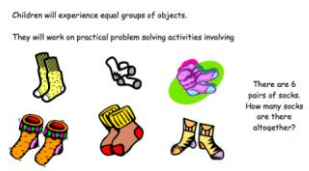
Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?

Multiplication

Vocabulary

Part, whole, groups of, lots of.

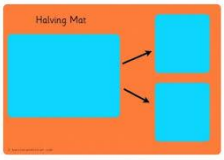




Division

Vocabulary

Part, whole, share, share equally, one each, two each..., group, groups of, lots of.



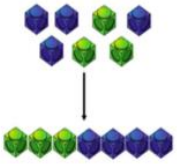
Year 1

Addition

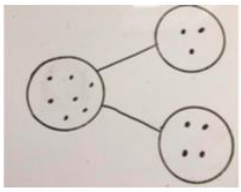
Vocabulary

Part, whole, addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

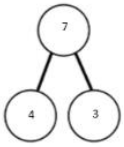
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).



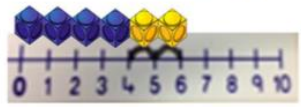
Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.

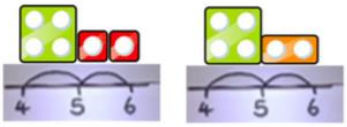


4 + 3 = 7
Four is a part, 3 is a part and the whole is seven.

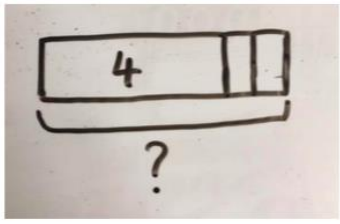


Counting on using number lines using cubes or Numicon.





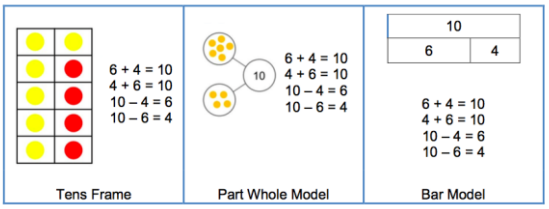
A bar model which encourages the children to count on, rather than count all.

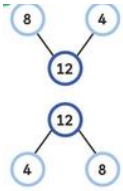


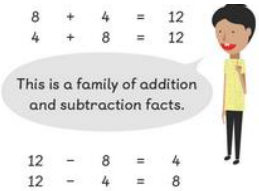
The abstract number line:
What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2?
4 + 2

Learn number bonds to 20 and demonstrate related facts

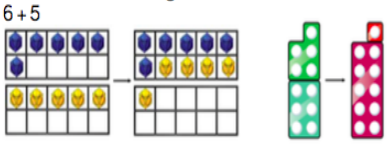
Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.



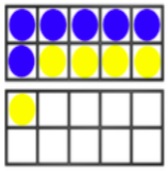




Regrouping to make 10; using ten frames and counters/cubes or using Numicon.
6 + 5



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

6 + □ = 11
6 + 5 = 5 + □
6 + 5 = □ + 4

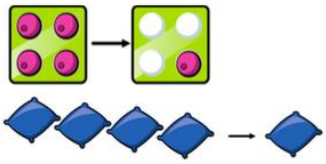
Subtraction

Vocabulary

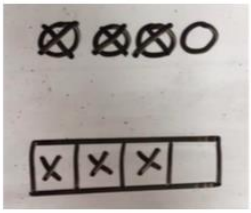
Part, whole, subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

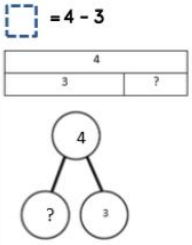
4 – 3 = 1



Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



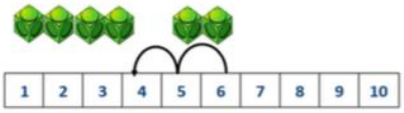
4- 3 =



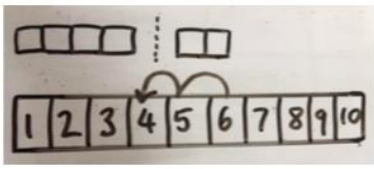
Subtraction by counting back

Counting back (using number lines or number tracks) children start with 6 and count back 2.

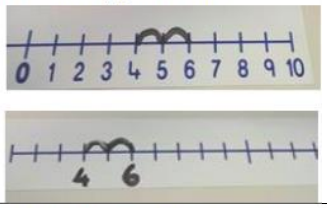
6 – 2 = 4



Children to represent what they see pictorially e.g.

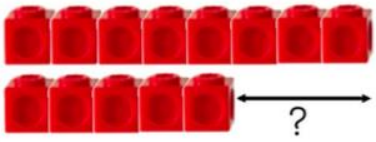


Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line

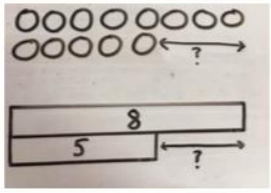


Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.

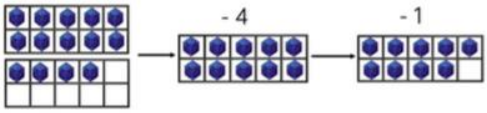


Find the difference between 8 and 5.

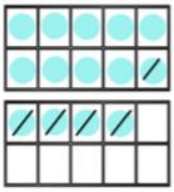
8 – 5, the difference is ☐

Children to explore why
9 - 6 = 8 – 5 = 7 – 4 have the same difference.

Making 10 using ten frames.
14 – 5



Children to present the ten frame pictorially and discuss what they did to make 10.

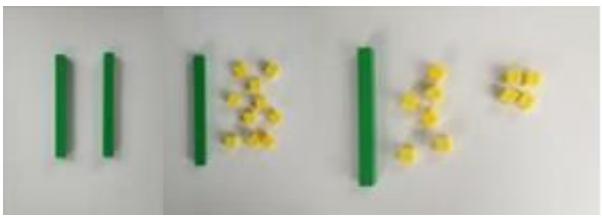


Children to show how they can make 10 by partitioning the subtrahend.

14 - 5 = 9
4 1

14 – 4 = 10
10 – 1 = 9

When subtracting using Dienes children should be taught to regroup a ten rod for 10 ones and then subtract from those ones

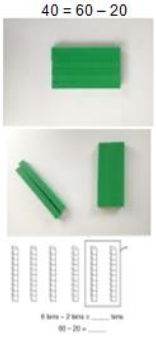


20 – 4 = 16

Subtracting multiples of 10

Using the vocabulary of 1 ten, 2 tens etc alongside 10, 20, 30 Is very important here as pupils need to understand that it is a 10 not a 1 that is being taken away

40 = 60 – 20



38 – 10 = 28

Multiplication

Vocabulary

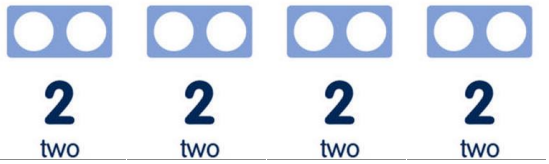
Part, whole, ones, groups, lots of, doubling, repeated addition, groups of, lots of, times, columns, rows, longer, bigger, higher etc and times as (big, long, wide …etc)

Counting in multiples of 2, 5 and 10 from zero

Children should count the number of groups on their fingers as they are skip counting.

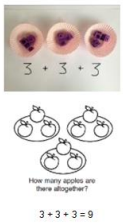
4 groups of 2 = 8

2 × 4 = 8



When moving to pictorial/written calculations the vocabulary is important

This image represents two groups of 4 or 4 twice



Solving multiplication problems using repeated addition

Division

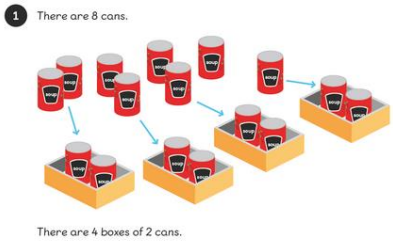
Vocabulary

Part, whole, share, share equally, one each, two each…, group, groups of, lots of, array

Pupils should be taught to divide through working practically and the sharing should be shown below the whole to familiarize children with the concept of the whole.

The language of whole and part part should be used.

8 ÷ 4 = 2



Year 2

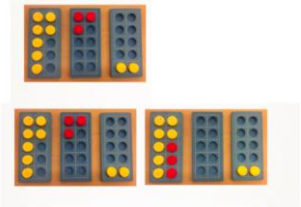
Addition

Vocabulary

Part, whole, +, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, tens, ones, partition

Near multiple of 10, tens boundary, more than, one more, two more… ten more…

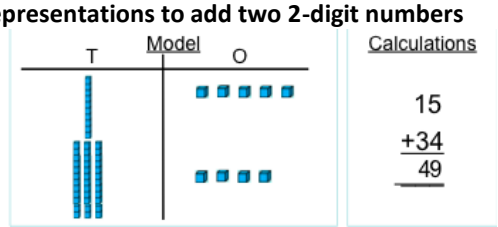
Using concrete objects and pictorial representations to add 3 single digit numbers.



7+3+2 = leads to 10 + 2 =

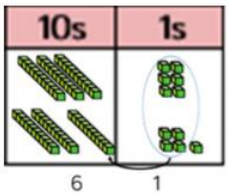
Using concrete objects and pictorial representations to add a 2 digit number and ones and tens.

Using concrete objects and pictorial representations to add two 2-digit numbers

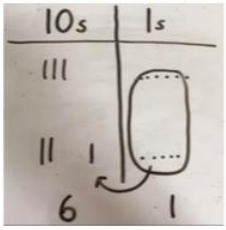


Leading to:

TO + TO using base 10. Continue to develop understanding of partitioning and place value.
36 + 25



Chidlren to represent the base 10 in a place value chart.



Looking for ways to make 10.

36 + 25=

1 5

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

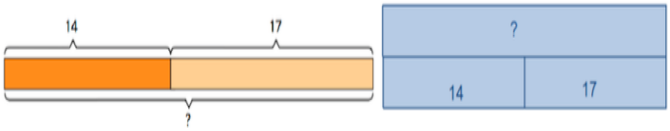
Formal method:

36
+25
61
1

Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

Helen has 14 bread-sticks. Her friend has 17. How many do they have altogether?



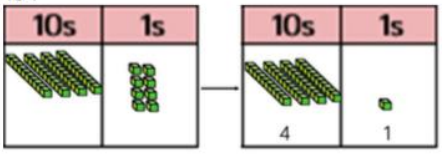
Subtraction

Vocabulary

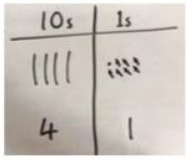
Part, whole, Subtraction, subtract, take away, difference, difference between, minus
Tens, ones, partition
Near multiple of 10, tens boundary, Less than, one less, two less… ten less…

Using concrete objects and pictorial representations to subtract a 1-digit number from 2-digit number.

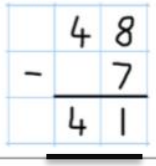
Column method using base 10.
48-7



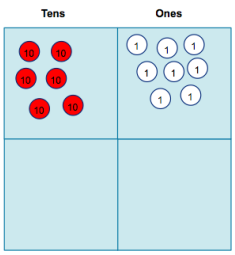
Children to represent the base 10 pictorially.



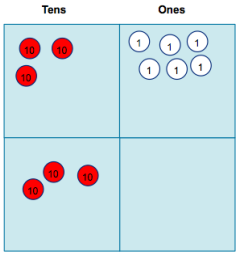
Column method or children could count back 7.



Using concrete objects and pictorial representations to subtract a 10s number from 2 digit number.

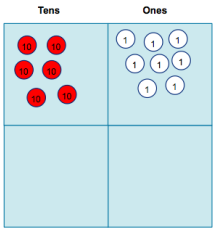


68
-30

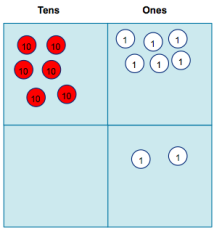


68
-30

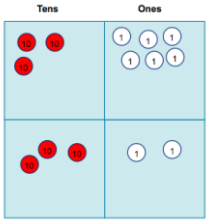
Using concrete objects and pictorial representations to subtract a 2-digit number from 2 digit number.



68
-32

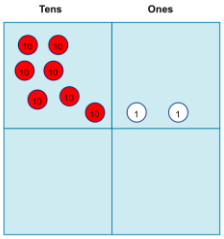


68
-32

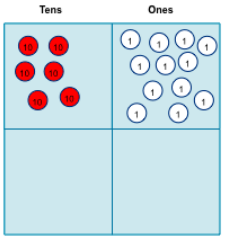


68
-32

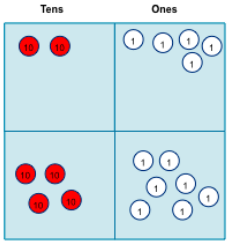
Greater Depth:



72
-47



~~7~~2
-47



~~7~~2
-47
25

Recognise and use the inverse relationship between addition and subtraction

76	
23	?

?	
23	53

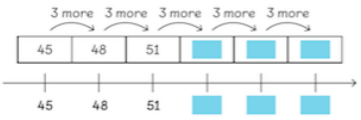
Use to check inverse calculations

Multiplication

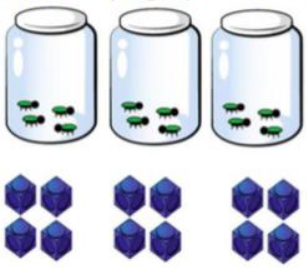
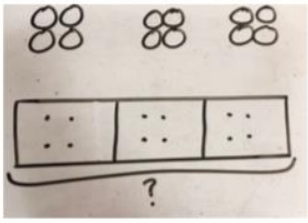
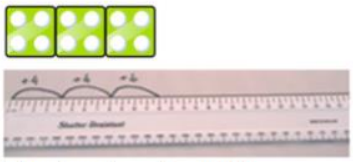
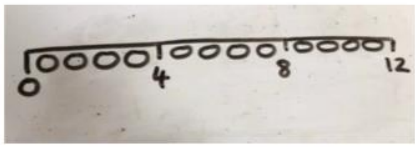
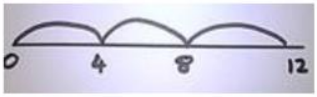
Vocabulary

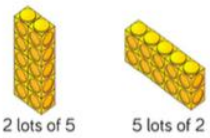
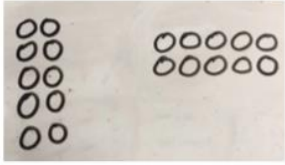
Part, whole, multiple, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows

Skip counting in multiples of 2, 3, 5, 10 from 0



Recall and use multiplication facts for the multiplication tables 2, 5 and 10.

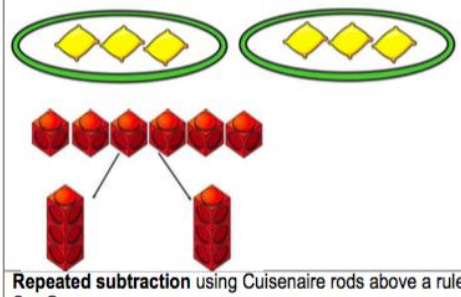
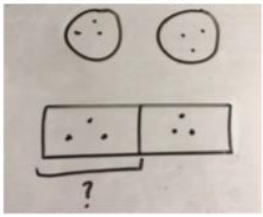
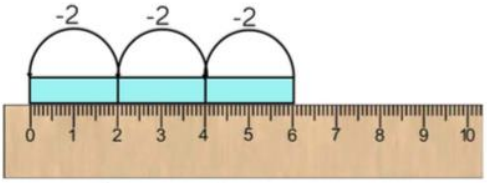
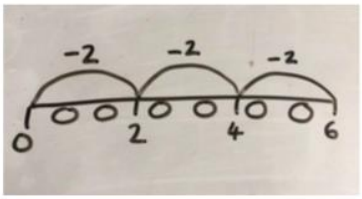
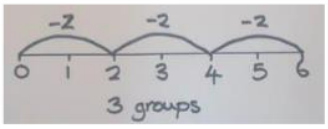
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model.  	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:  	Abstract number line showing three jumps of four. 3 × 4 = 12 

Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

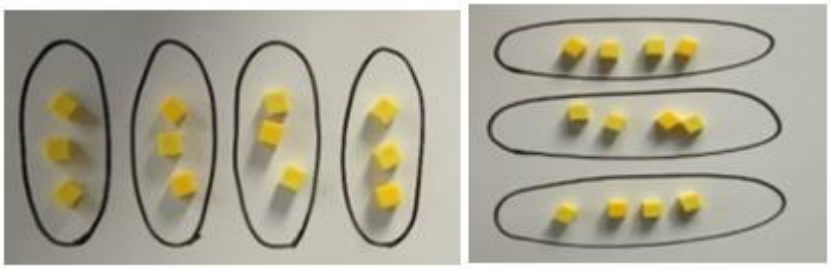
Division

Vocabulary

Part, whole, group in pairs, 3s … 10s etc, equal groups of, divide, ÷, divided by, divided into, remainder

Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

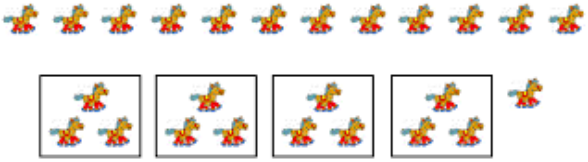
Solve division problems in context using arrays



I can solve division as grouping.

Put 10 buns in groups of 2.
How many plates are there?

Greater Depth with remainders



13 ÷ 4 = 3 Remainder 1

Year 3

Addition

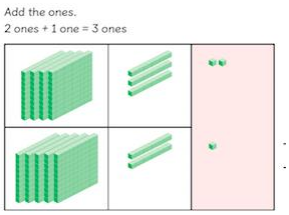
Vocabulary

Part, whole, hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2

Add two three-digit numbers.

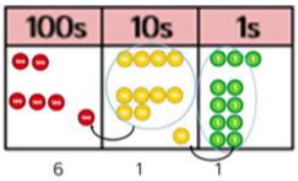
Children need to use equipment first to support their understanding of place value.

Children to progress gradually to three digit + three digit starting without carrying and gradually moving towards carrying.

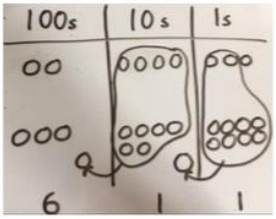


	h	t	o
	4	3	2
+	5	2	1
			3

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



Chidren to represent the counters in a place value chart, circling when they make an exchange.



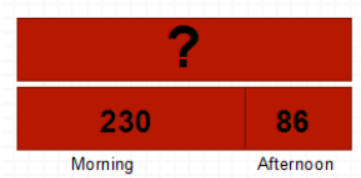
$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

Bar Model to support understanding of problem solving:

A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening . How many balloons did he sell in all?



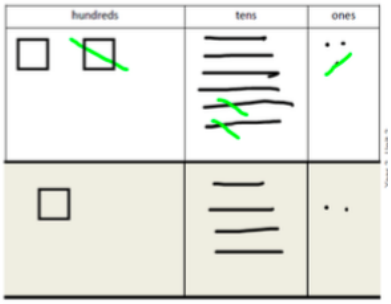
Subtraction

Vocabulary

Part, whole, hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2

Subtract up to 3 digits from 3 digits.

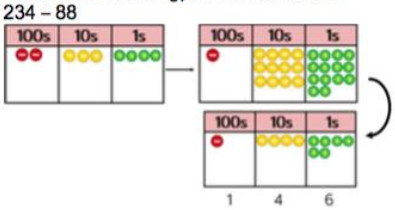
Very important for children to use dienes equipment along with a place value chart to support.



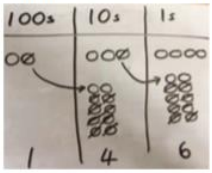
263 – 121= 142

Only when secure with the method should exchanging be introduced.

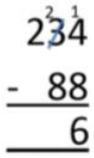
Column method using place value counters.
234 – 88



Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal colum method. Children must understand what has happened when they have crossed out digits.



Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

315	
185	?

315 – 185 = ?
185 + ? = 315

?	
185	315

185 + 315 = ?
? – 185 = 315

Multiplication

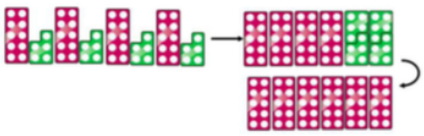
Vocabulary

Part, whole, multiple, partition, short multiplication and inverse

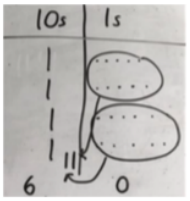
Children should be able to recall the 2, 5, 10, 3, 4 and 8 times tables.

Multiply a two-digit number by a one digit.

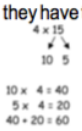
Partition to multiply using Numicon, base 10 or Cuisenaire rods.
4 × 15



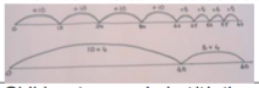
Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

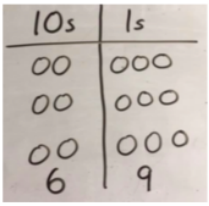


A number line can also be used



Formal column method with place value counters (base 10 can also be used.) 3 × 23

Children to represent the counters pictorially.



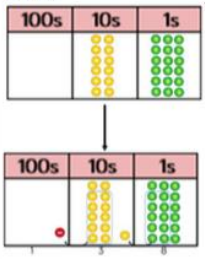
Children to record what it is they are doing to show understanding.

3 × 23
20 3

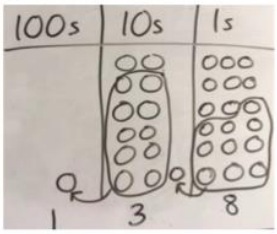
3 × 20 = 60
3 × 3 = 9
60 + 9 = 69

23
× 3
69

Formal column method with place value counters.
6 x23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

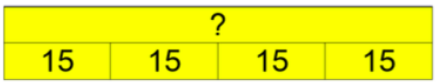
6 x 23 =

23
× 6
138
1 1

Using the bar to solve multiplication problems.

4 children go to the cinema. They each pay £15. How much do they spend altogether?

Whole unknown

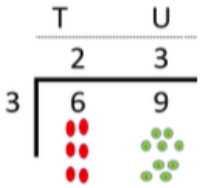


Division

Vocabulary

Part, whole, See Y1 and Y2 and Inverse, remainder

Dividing using short division.

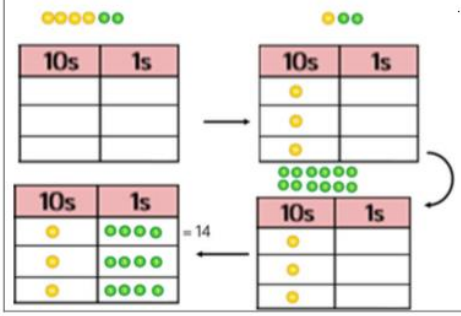
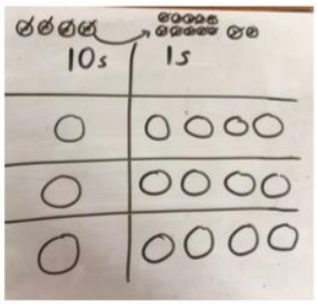


Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:

· *How many 3's in 6? = 2, and record it above the **6 tens**.*

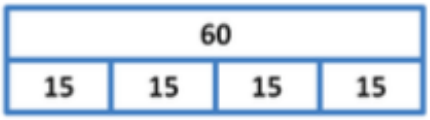
· *How many 3's in 9? = 3, and record it above the **9 ones**.*

Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 42÷3), and be taught to 'carry' the remainder onto the next digit.

Sharing using place value counters. 42 ÷ 3 = 14	Children to represent the place value counters pictorially.	Children to be able to make sense of the place value counters and write calculations to show the process.
		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Using the bar to aid the solving of division problems – grouping and sharing

60 ÷ 4 = 15



'60 in four equal parts'

28 ÷ 7 = 4

'How many 7s in 28?'

Year 4

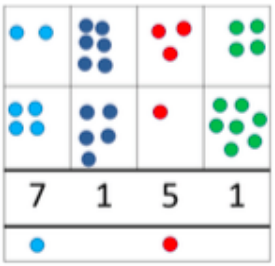
Addition

Vocabulary

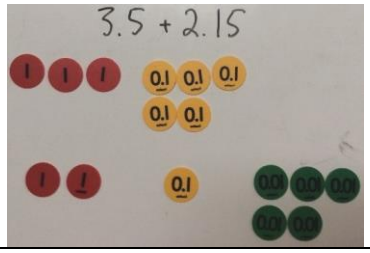
Part, whole, add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Adding numbers with up to 4 digits.

Again this should start with the children using dienes to support them with lots of discussion about the value of each digit.



2634
+4517
7151



Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

This is not a form of getting the correct answer but helping to guide children to the correct operation.

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

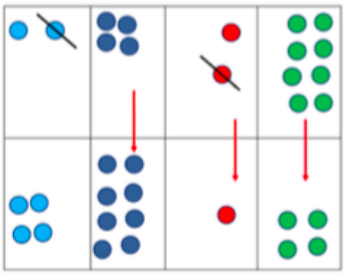
Subtraction

Vocabulary

Part, whole, subtract, takeaway, less, minus, decrease, fewer, difference, how many less to make..? how much less? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many fewer? Equals sign, is the same as.

To subtract with numbers up to four digits including exchanging when children are secure.

Children need to use place value counters to support their learning.

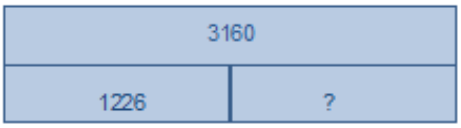


5 1 2 1
~~6~~2~~3~~2
- 4814
1418

Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?



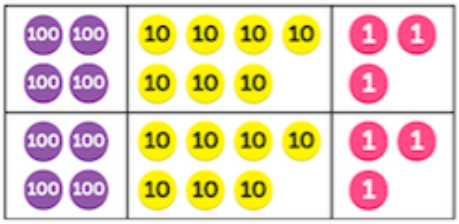
Multiplication

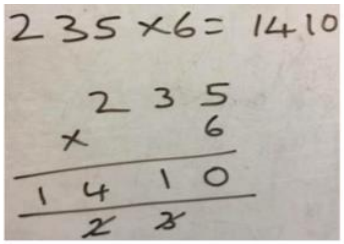
Vocabulary

Part, whole, Factor, product

Children to know all times tables to 12 x 12.

Children multiplying both two and three digits by a one digit number using place value counters.





Multiplying using the bar.

A computer costs 5 times as much as a television. The television costs £429.

How much does the computer cost?

Cost of the computer

Division

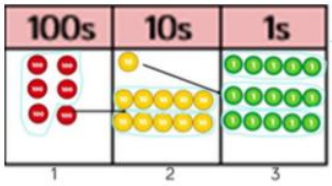
Vocabulary

Part, whole, see years 1-3, divide, divided by, divisible by, divided into, share between, groups of factor, factor pair, multiple, times as (big, long, wide …etc), equals, remainder, quotient, divisor and inverse

Dividing up to three digit numbers by a one digit number using short division.

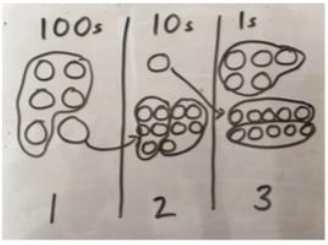
Only when the children are secure with dividing a two digit number should they move onto a 3 digit number.

Short division using place value counters to group.
615÷ 5

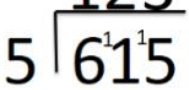


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

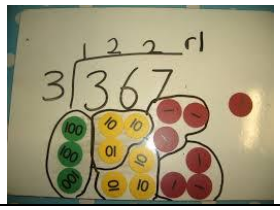
Represent the place value counters pictorially.

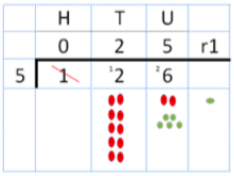


Children to the calculation using the short division scaffold.



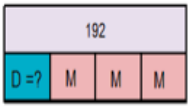
With remainders





Dividing using the bar.

Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have?



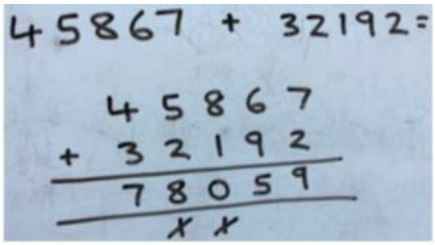
Year 5

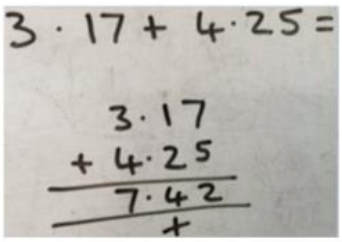
Addition

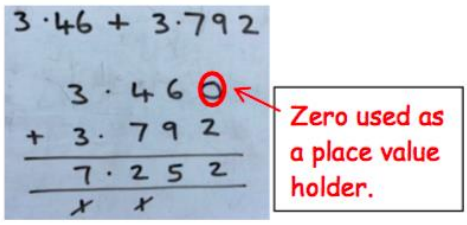
Vocabulary
Part, whole, tens of thousands boundary,
Also see previous years

Adding numbers with more than 4 digits including decimals

Using place value charts are key to this as well as place value counters to help with the decimals.





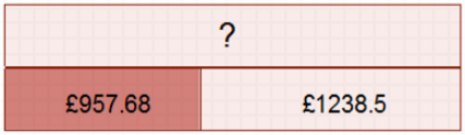


Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

This is not a form of getting the correct answer but helping to guide children to the correct operation.

MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether?



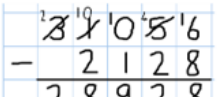
Subtraction

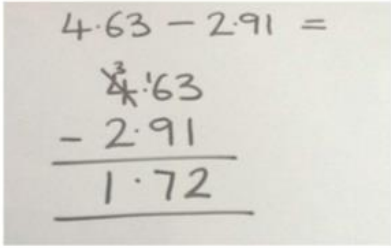
Vocabulary
Part, whole, tens of thousands boundary,
Also see previous years

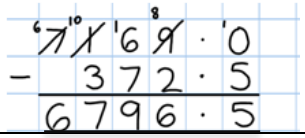
Subtract with at least four digit numbers including two decimal places.

Include money, measures and decimals ensuring that children do this practically before the abstract.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.



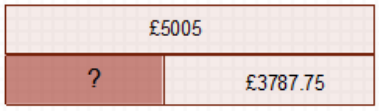




Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

A whole to Lapland costs £5005 for a family of four, the Smith's have only saved £3787.75, how much money do they still need to find?



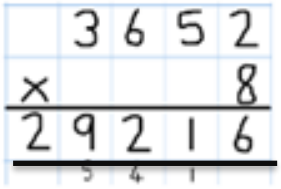
Multiplication

Vocabulary

Part, whole, cube numbers, prime numbers, square numbers, common factors, prime number, prime factors and composite numbers

Multiplying up to four digit numbers by two digits using long multiplication.

*Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the reasonableness of their answer.*



(1234 × 6)

(1234 × 10)

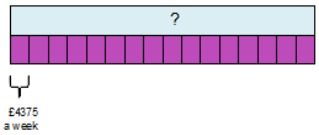
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

Answer: 3224

Using the bar to support multiplication.

The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks?



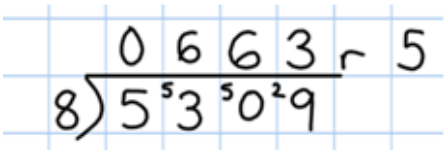
Division

Vocabulary see year 4

Part, whole, common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of

Diving with up to four digit numbers by one digit including numbers where remainders are left.

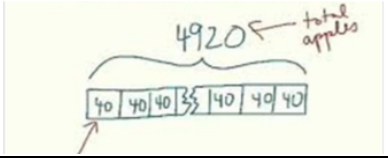
Short division with remainders: Now give rise to remainder answers, solving context, where **pupils consider** that pupils are introduced to examples that division needs to have a real life problem **express it,** ie. as a fraction, a decimal, **the meaning of the remainder and <u>how</u> to** upon the context of the problem. or as a rounded number or value , depending



Using the bar to support division problems.

<u>Bar Model to support understanding of problem solving:</u>

Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need?



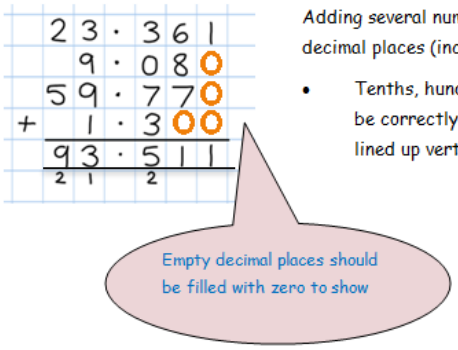
Year 6 (supporting transition into Year 7)

Addition

Vocabulary

Part, whole, See previous years

Adding decimal several numbers with up to three places.



Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Adding using the bar.

Jack went on holiday. His flight cost £70.50, the hotel £1295 and spending money £427.89. How much did Jack spend on his holiday?

?		
£70.50	£427.89	£1295

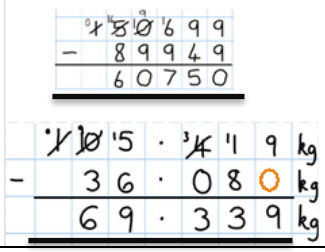
Subtraction

Vocabulary

Part, whole, See previous years

Subtracting with increasingly large and more complex numbers and decimal values.

Very important to use in a range of contexts- measures and money.



Using the bar for subtraction.

Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save?

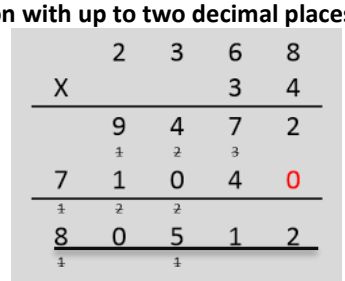
£6450		
£4885.87	£150	?

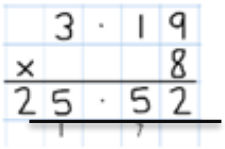
Multiplication

Vocabulary

Part, whole, See previous years and common factor

Short and long multiplication with up to two decimal places.





Using the bar to help with multiplication.

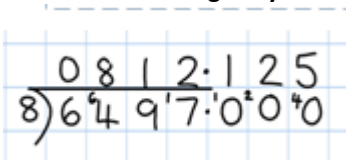
If 5 friends went on holiday and each paid £579.75 what was the total cost of the holiday?

Division

Vocabulary

see years 4 and 5 Part, whole, long division

Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

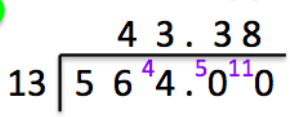


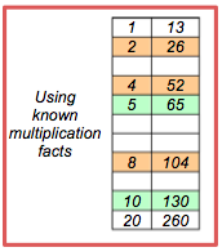
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Long division this is for when dividing by two digit numbers.

Division

564 ÷ 13



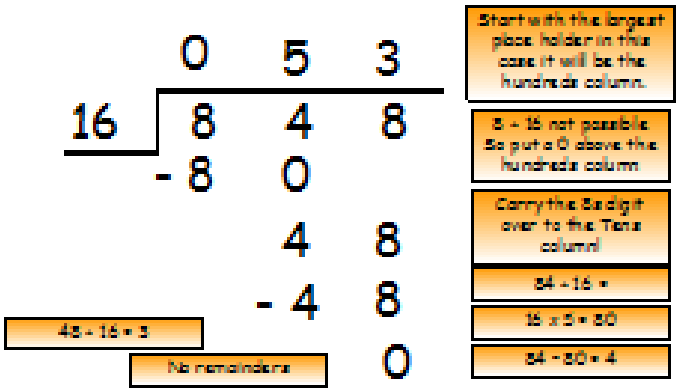


564 ÷ 13

$= 43 \text{ r } 5 = 43\frac{5}{13} = 43.4$ (to 1dp)

Try this equation: 848 ÷ 16

Approximation 800 ÷ 16 = 50



Using the bar to help divide.

Paul and David hire a car together at a cost of £297.50. Paul pays 6 times more than David. How much does David pay?